Period 3, Oct 28, 2024

$$h(x) = \ln(\ln(2x)) = \ln(\ln u) = \ln L = y$$

$$\frac{d}{dx}[\ln u] = \frac{u'}{u}$$

$u = 2x$ $\qquad$ $L = \ln u$ $\qquad$ $\frac{dy}{dL} = \frac{1}{L}$

$\frac{du}{dx} = 2$ $\qquad$ $\frac{dL}{du} = \frac{1}{u}$

$$\frac{du}{dx}\cdot\cancel{\frac{dL}{du}}\cdot\frac{dy}{\cancel{dL}} = \frac{dy}{du}$$

$$2\cdot\frac{1}{u}\cdot\frac{1}{L} = \cancel{2}\cdot\frac{1}{\cancel{2}x}\cdot\frac{1}{\ln u} = = \frac{1}{x\ln 2x}$$

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b) $y = \sqrt[3]{\frac{(x-2)^3(x^2+5)}{(2x+3)^3}} \Rightarrow \ln y = \ln\left[\frac{(x-2)^3(x^2+5)}{(2x+3)^3}\right]^{\frac{1}{3}} \Rightarrow \ln y = \frac{1}{3}\left[\ln\frac{(x-2)^3(x^2+5)}{(2x+3)^3}\right.$

$$\ln y = \frac{1}{3}\left[\ln(x-2)^3 + \ln(x^2+5) - \ln(2x+3)^3\right]$$

$$\ln y = \frac{1}{3}\left[3\ln(x-2) + \ln(x^2+5) - 3\ln(2x+3)\right]$$

$$\ln y = 1\ln(x-2) + \frac{1}{3}\ln(x^2+5) - 1\ln(2x+3)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x-2}\cdot 1 + \frac{1}{3}\cdot\frac{1}{x^2+5}\cdot 2x - \frac{1}{2x+3}\cdot 2$$

$$y\cdot\cancel{\frac{1}{y}}\frac{dy}{dx} = \left(\frac{1}{x-2} + \frac{2x}{3(x^2+5)} - \frac{2}{2x+3}\right)\cdot y \Rightarrow$$

$$\frac{dy}{dx} = \left(\frac{1}{x-2} + \frac{2x}{3(x^2+5)} - \frac{2}{2x+3}\right)\left(\sqrt[3]{\frac{(x-2)^3(x^2+5)}{(2x+3)^3}}\right)$$

e) $h(x) = x^{\pi}$ $\Rightarrow \underbrace{h'(x) = \pi x^{\pi-1}}$

$y = x^{3.14159\ldots}$

$\frac{dy}{dx} = (3.14159\ldots)\, x^{3.14159\ldots - 1}$

$y = x^3$

$\frac{dy}{dx} = 3x^{3-1} = 3x^2$

g) $g(x) = \log_2 \frac{1}{x}$

$y = \log_2 \frac{1}{x} \Rightarrow 2^y = \frac{1}{x} \Rightarrow \ln 2^y = \ln \frac{1}{x} = \ln 1 - \ln x = 0 - \ln x$

$\Downarrow$

$y \ln 2 = \ln 1 - \ln x$

$\frac{dy}{dx} \cdot \ln 2 = 0 - \frac{1}{x}$

$\frac{1}{\ln 2} \cdot \frac{dy}{dx} \cancel{\cdot \ln 2} = -\frac{1}{x} \cdot \frac{1}{\ln 2} = \frac{-1}{x \ln 2}$

11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$

g) $g(x) = \log_2 \frac{1}{x}$

$y = \log_2 x^{-1} \Rightarrow \frac{dy}{dx} = \frac{-1x^{-2}}{(\ln 2)\, x^{-1}} = \frac{\frac{-1}{x^2}}{\frac{\ln 2}{x}} = \frac{-1}{x^{\cancel{2}}} \cdot \frac{\cancel{x}}{\ln 2} = \frac{-1}{x \ln 2}$

0. Find $\frac{d^2y}{dx^2}$ if $y^3 + y = 2\cos x$

$$3y^2\frac{dy}{dx} + \frac{dy}{dx} = 2(-\sin x) = -2\sin x$$

$$\frac{\frac{dy}{dx}\cancel{(3y^2+1)}}{\cancel{(3y^2+1)}} = \frac{-2\sin x}{(3y^2+1)}$$

$$\frac{dy}{dx} = \frac{-2\sin x}{(3y^2+1)}$$

$$\frac{d^2y}{dx^2} = \frac{-2(\cos x)(3y^2+1) - (-2\sin x)(6y\frac{dy}{dx} + 0)}{(3y^2+1)^2}$$

$$\frac{d^2y}{dx^2} = \frac{-2(\cos x)(3y^2+1) + 2(\sin x)\left(6y\left(\frac{-2\sin x}{3y^2+1}\right)\right)}{(3y^2+1)^2}$$

a) $y = (\sin x)^x$

$$\ln y = \ln(\sin x)^x = \underbrace{x}\,\underbrace{\ln(\sin x)}$$

$$\frac{1}{y}\frac{dy}{dx} = 1 \cdot \ln(\sin x) + x \cdot \frac{1}{\sin x} \cdot \cos x = \ln(\sin x) + x \cdot \frac{\cos x}{\sin x}$$

$$\cancel{y} \cdot \frac{1}{\cancel{y}}\frac{dy}{dx} = [\ln(\sin x) + x\cot x] \cdot y$$

$$\frac{dy}{dx} = [\ln(\sin x) + x\cot x][(\sin x)^x]$$

$$y = 3x$$

$$\frac{dy}{dx} = 3$$

3.Find $g'(x)$ if $g(x) = \underbrace{2^x}\cdot\underbrace{\log_3\sqrt{x-1}}$.

$g(x) = 2^x \log_3\sqrt{x-1}$

$g(x) = F(x)\cdot h(x)$

$g'(x) = F'(x)\cdot h(x) + F(x)\cdot h'(x)$

$g'(x) = (\ln 2)(2^x)\cdot\log_3\sqrt{x-1}$

$+$

$2^x \cdot \frac{1}{2(x-1)\ln 3}$

$y = 2^x$

12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$

$\ln y = \ln 2^x = x\ln 2$

$y\cdot\frac{1}{y}\frac{dy}{dx} = \ln 2\cdot y$

$$\boxed{\frac{dy}{dx} = (\ln 2)(2^x)}$$

$y = \log_3\sqrt{x-1} = \log_3(x-1)^{\frac{1}{2}}$

$\ln 3^y = \ln(x-1)^{\frac{1}{2}}$

$y\cdot\ln 3 = \frac{1}{2}\ln(x-1)$

$\frac{dy}{dx}\cdot\ln 3 = \frac{1}{2}\cdot\frac{1}{x-1}\cdot 1$

$\frac{1}{\cancel{\ln 3}}\cdot\frac{dy}{dx}\cdot\cancel{\ln 3} = \frac{1}{2(x-1)}\cdot\frac{1}{\ln 3}$

$\frac{dy}{dx} = \frac{1}{2(x-1)\ln 3}$

11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$

$$\lim_{x\to 2}\frac{x^2-4}{x-2} =$$

$\lim \frac{F(x)}{g(x)} = \frac{0}{0}, \pm\frac{\infty}{\infty}$

$\lim \frac{F(x)}{g(x)} = \lim \frac{F'(x)}{g'(x)}$

$\lim_{x\to 2}\frac{x^2-4}{x-2} = \frac{2^2-4}{2-2} = \frac{4-4}{2-2} = \frac{0}{0} \Rightarrow \lim_{x\to 2}\frac{x^2-4}{x-2} = \lim_{x\to 2}\frac{2x}{1} = 2\cdot 2 = 4$

Evaluate $\lim_{x \to 0} \frac{e^{2x} - 1}{x}$

$$\lim_{x \to 0} \frac{e^{2x}-1}{x} = \frac{e^{2\cdot 0}-1}{0} = \frac{e^{0}-1}{0} = \frac{1-1}{0} = \frac{0}{0} = \emptyset = \frac{RSN}{RSN}$$

$$\lim_{x \to 0} \frac{e^{2x}-1}{x} = \lim_{x \to 0} \frac{2e^{2x}}{1} = \frac{2\cdot e^{2\cdot 0}}{1} = 2\cdot e^{0} = 2\cdot 1 = 2$$

$$\lim_{x \to \infty} \frac{\ln x}{x} = \frac{\ln \infty}{\infty} = \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{\ln x}{x} = \lim_{x \to \infty} \frac{\frac{1}{x}}{1} = \frac{1}{\infty} = 0$$

$$\lim_{x \to 0} \frac{F(x)}{g(x)} = \text{indeterminate}$$

$$\lim_{x \to 0} \frac{F(x)}{g(x)} = \lim_{x \to 0} \frac{F'(x)}{g'(x)}$$

Example 3

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$$

$$\Rightarrow \lim_{x\to 0}\frac{(\sqrt{1+x}-1)}{x}\cdot\frac{(\sqrt{1+x}+1)}{(\sqrt{1+x}+1)}$$

$$\lim_{x\to 0}\frac{\cancel{1}+x+\cancel{\sqrt{1+x}}-\cancel{\sqrt{1+x}}\cancel{-1}}{x(\sqrt{1+x}+1)} = \lim_{x\to 0}\frac{\cancel{x}}{\cancel{x}(\sqrt{1+x}+1)} = \frac{1}{\sqrt{1+0}+1} = \frac{1}{1+1}$$

$$= \frac{1}{2}$$

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x} = \frac{\sqrt{1+0}-1}{0} = \frac{\sqrt{1}-1}{0} = \frac{1-1}{0} = \frac{0}{0} = \emptyset$$

$$\lim_{x\to 0}\frac{(1+x)^{\frac12}-1}{x} = \frac{\frac12(1+x)^{-\frac12}\cdot 1-0}{1} = \frac{\frac{1}{2\sqrt{1+x}}}{1} = \frac{1}{2\sqrt{1+0}} = \frac{1}{2\sqrt{1}} = \frac{1}{2}$$

$$\lim_{x\to-\infty}\frac{x^2}{e^{-x}} = \frac{(-\infty)^2}{e^{-(-\infty)}} = \frac{\infty^2}{e^{\infty}} = \frac{\infty}{\infty}$$

$$\lim_{x\to-\infty}\frac{x^2}{e^{-x}} = \lim_{x\to-\infty}\frac{2x}{-e^{-x}} = \lim_{x\to-\infty}\frac{2}{e^{-x}} = \frac{2}{e^{-(-\infty)}} = \frac{2}{e^{\infty}} = \frac{2}{RBN} = 0$$

$$\frac{2(-\infty)}{-e^{-(-\infty)}} = \frac{-\infty}{-\infty}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b}\cdot\frac{d}{c}$$

$$\frac{\sin\frac{1}{x}}{1}\cdot\frac{x}{1} = \frac{\frac{\sin\frac{1}{x}}{1}}{\frac{1}{x}} = \frac{\sin\frac{1}{x}}{\frac{1}{x}}$$

$$y = x^{-1} = \frac{1}{x}$$

$$\frac{dy}{dx} = -1x^{-1-1} = -1x^{-2} = \frac{-1}{x^2}$$

$$\lim_{x\to\infty}\left(x\sin\frac{1}{x}\right) = \infty\cdot\sin\frac{1}{\infty} = \infty\cdot \sin RSN = \infty\cdot 0$$

$$\lim_{x\to\infty} x\sin\frac{1}{x} = \lim_{x\to\infty}\frac{\sin\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty}\frac{\left(\cos\frac{1}{x}\right)\cdot\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{\left(-\frac{1}{x^2}\right)}} = \cos\frac{1}{\infty} = \cos 0 = 1$$

$$1 = \lim_{x\to\infty} x^{1/x} = \infty^{\frac{1}{\infty}} = \infty^{0}$$

$$1 = y = \lim_{x\to\infty} x^{\frac{1}{x}}$$

$$\ln y = \ln\left[\lim_{x\to\infty} x^{\frac{1}{x}}\right] = \lim_{x\to\infty}\ln x^{\frac{1}{x}} = \lim_{x\to\infty}\frac{1}{x}\cdot\ln x = \lim_{x\to\infty}\frac{\ln x}{x} = \frac{\infty}{\infty}$$

$$\ln y = \lim_{x\to\infty}\frac{\ln x}{x} = \lim_{x\to\infty}\frac{\frac{1}{x}}{1} = \frac{1}{\infty} = 0 \Rightarrow \ln y = 0 \Rightarrow e^{0} = y$$

$$1 = y$$

$$\ln y = 0 \Rightarrow e^{0} = y \Rightarrow 1 = y$$

$$\lim_{x \to 0^+} x^x = 0^0 = \text{indetorminate}$$

$$y = \lim_{x \to 0} x^x$$

$$\ln y = \lim_{x \to 0} \ln x^x$$

$$\ln y = \lim_{x \to 0} \ln x^x = \lim_{x \to 0} x \cdot \ln x = \lim_{x \to 0} \frac{\ln x}{\frac{1}{x}} = \lim_{x \to 0} \frac{\frac{1}{x}}{\frac{-1}{x^2}}$$

$$= \lim_{x \to 0^+} \frac{\cancel{\frac{1}{x}}}{\left(\frac{-1}{x}\right)\cancel{\left(\frac{1}{x}\right)}}$$

$$\lim_{x \to 0^+} \frac{1}{\frac{-1}{x}} = \frac{1}{\frac{-1}{RSN}}$$

$$\lim_{x \to 0^+} \frac{1}{-\infty} = 0$$

$$\ln y = 0 \Rightarrow e^0 = y \Rightarrow 1 = y$$

$$\lim_{x \to c} F(x) = -14 \qquad \lim_{x \to c} g(x) = -10$$

$$\lim_{x \to c} \left[F^2(x) - \frac{1}{2}(g(x))\right] = [-14]^2 - \frac{1}{2}(-10) = 196 + 5$$

$$= 201$$

$$\lim_{x \to c} F(x) = -14 \quad \lim_{x \to c} g(x) = -10$$

$$\lim_{x \to c} [5F(x) - g^2(x)] = 5(-14) - (-10)^2 = -70 - 100$$

$$-170$$

$$\lim_{x\to 0^-}\left(x^8 - \frac{1}{x}\right) = 0^8 - \frac{1}{-RSN} = 0 + \frac{1}{RSN} = \infty = \emptyset$$

$$\lim_{x\to 0^-}\left(x^5 - \frac{1}{x}\right) = 0^8 - \frac{1}{-RSN} = 0 + \frac{1}{RSN} = \infty = \emptyset$$

$$\lim_{x\to 0}\frac{(1-\cos 2x)}{3x}\cdot\frac{(1+\cos 2x)}{(1+\cos 2x)} = \lim_{x\to 0}\frac{1-\cancel{\cos 2x}+\cancel{\cos 2x}-\cos^2 2x}{3x(1+\cos 2x)}$$

$$\lim_{x\to 0}\frac{1-\cos^2 2x}{3x(1+\cos 2x)} = \lim_{x\to 0}\frac{\sin^2 2x}{3x(1+\cos 2x)}$$

$$\lim_{a\to 0}\frac{\sin a}{a} = \lim_{a\to 0}\frac{a}{\sin a} = 1$$

$$\lim \frac{\frac{2}{3}\cdot\sin^2 2x}{\frac{2}{3}\cdot 3x(1+\cos 2x)} =$$

$\sin 0 = 0$

$\cos 0 = 1$

$$\lim_{x\to 0}\frac{2}{3}\underbrace{\boxed{\frac{\sin 2x}{2x}}}_{1}\cdot\underbrace{\boxed{\frac{\sin 2x}{(1+\cos 2x)}}}_{\frac{0}{2}} = \frac{2}{3}\cdot 1\cdot\frac{0}{2} = 0$$

$$\lim_{x\to 0}\frac{1-\cos 2x}{3x} = \lim_{x\to 0}\frac{1-(1-2\sin^2 x)}{3x} = \lim_{x\to 0}\frac{2\sin x\cdot\sin x}{3x}$$

$$\frac{2}{3}\cdot 1\cdot 0 = 0$$

Double Angle Identities

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

$$\lim_{x\to 0} \frac{1-\cos^2 x}{x} = \lim_{x\to 0} \frac{\sin^2 x}{x} = \lim_{x\to 0} \boxed{\frac{\sin x}{x}} \cdot \boxed{\frac{\sin x}{1}}$$

$$1 \cdot 0 = 0$$

$$\lim_{\Delta x\to 0} \frac{\left(\sqrt{x+\Delta x}-\sqrt{x}\right)}{\Delta x}\cdot\frac{\left(\sqrt{x+\Delta x}+\sqrt{x}\right)}{\left(\sqrt{x+\Delta x}+\sqrt{x}\right)} = \lim_{\Delta x\to 0} \frac{\cancel{x}+\Delta x+\cancel{\sqrt{x}\sqrt{x+\Delta x}}-\cancel{\sqrt{x}\sqrt{x+\Delta x}}-\cancel{x}}{\Delta x\left(\sqrt{x+\Delta x}+\sqrt{x}\right)}$$

$$\lim_{\Delta x\to 0} \frac{\cancel{\Delta x}}{\cancel{\Delta x}\left(\sqrt{x+\Delta x}+\sqrt{x}\right)}$$

$$\lim_{\Delta x\to 0} \frac{1}{\sqrt{x+\Delta x}+\sqrt{x}} = \frac{1}{\sqrt{x+0}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$\lim_{x\to 0} \frac{15x\cdot\sin 5x}{15x\cdot\sin 3x}$$

$$\lim_{x\to 0} \frac{5\cdot 3x\cdot\sin 5x}{3\cdot 5x\cdot\sin 3x}$$

$$1 \qquad 1$$

$$\frac{5}{3}\cdot 1\cdot 1 = \frac{5}{3}$$

$$\lim_{a\to 0}\frac{\sin a}{a} = 1 = \lim_{a\to 0}\frac{a}{\sin a}$$

$$\lim_{x\to 0} \frac{12x\,\sin 4x}{12x\,\sin 3x} = \lim_{x\to 0} \frac{3x\cdot 4\cdot\sin 4x}{4x\cdot 3\,\sin 3x} = 1\cdot 1\cdot\frac{4}{3} = \frac{4}{3}$$

$$1 \qquad 1$$

$$\lim_{x\to 0} \frac{\sin 4x}{\sin 3x} = \frac{\sin 0}{\sin 0} = \frac{0}{0} = \emptyset \Rightarrow \lim_{x\to 0} \frac{\sin 4x}{\sin 3x} = \lim_{x\to 0}\frac{4\cos 4x}{3\cos 3x} = \frac{4\cdot 1}{3\cdot 1}$$

$$\cos 0 = 1$$

$$\lim_{a\to 0} \frac{\sin a}{a} = \lim_{a\to 0} \frac{a}{\sin a} = 1$$

$$\lim_{a\to 0} \frac{1-\cos a}{a} = 0$$

$$\lim_{x\to 0} \frac{3x^2(1+\cos 5x)}{(1-\cos 5x)(1+\cos 5x)} = \lim_{x\to 0} \frac{3x^2(1+\cos 5x)}{1+\cancel{\cos 5x}-\cancel{\cos 5x}-\cos^2 5x}$$

$$\lim_{x\to 0} \frac{3x^2(1+\cos 5x)}{1-\cos^2 5x} =$$

$$\lim_{x\to 0} \frac{3x^2(1+\cos 5x)\cdot 25}{\sin^2 5x \cdot 25}$$

$$\lim_{x\to 0} \frac{3\cdot 5x\cdot 5x\cdot(1+\cos 5x)}{25\cdot \sin 5x\cdot \sin 5x}$$

$1 \quad 1$

$1+1=2$

$$\lim_{x\to 0} \frac{3(1+\cos 5x)}{25} = \frac{3(1+\cos 0)}{25} = \frac{6}{25}$$

$$\lim_{x\to \pi} \tan\frac{5x}{3} = \lim_{x\to \pi} \frac{\sin\frac{5x}{3}}{\cos\frac{5x}{3}} = \frac{\sin\frac{5\pi}{3}}{\cos\frac{5\pi}{3}} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \cancel{-\frac{\sqrt{3}}{2}\cdot\frac{2}{1}}$$

$$= -\sqrt{3}$$

$$\lim_{m \to 0} \frac{\cos(m+\frac{\pi}{2})}{m} = \lim_{m \to 0} \frac{\cos m \cos\frac{\pi}{2} - \sin m \cdot \sin\frac{\pi}{2}}{m}$$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\cos\frac{\pi}{2} = 0$

$\sin\frac{\pi}{2} = 1$

$$\lim_{m \to 0} \frac{\cancel{(\cos m)(0)}^{0} - \sin m \cdot 1}{m}$$

$$\lim_{m \to 0} -\boxed{\frac{\sin m}{m}}_{1} = -1$$